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A New Low Bit-Rate Waveform Coding Using Extrapolation and Predictive Coding

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ABSTRACT

This paper describes a new waveform coding method using extrapolation and predictive coding techniques. The proposed method can provide 80% bit-rate reduction compared to linear PCM coding.

I. INTRODUCTION

Low bit-rate coding technique is one of key factors in speech communication and storage.^[1] This paper presents a new low bit-rate waveform coding method in which oversampling, predictive coding and extrapolation techniques are effectively combined to reduce bit-rate.

Oversampling technique is used to convert a input signal into highly correlated signal. Predictive coding provides low bit-rate encoding scheme for such a oversampled signal.

In addition to these techniques, extrapolation is introduced in a decoder. If a signal is band limited, we can reconstruct an original signal from a part of the signal by extrapolating the rest. Hence, we need transmit only a part of signal samples, not a whole samples. Applying the extrapolation to the decoder input signal, the proposed coding method can reduce the bit-rate by 80% over conventional linear PCM coding.

II. CONFIGURATION OF CODEC

Fig.1 shows a block diagram of the proposed coder and decoder, and Fig.2 shows the signal waveforms in each stage.

A. CODER

Let $x(n)$ be a band limited signal with band width $0 \leq f \leq f_b$, sampled at the sampling frequency $f_s = 2f_b$. By inserting $M-1$ zeros between each sample of $x(n)$, we obtain $u(m)$ with the sampling rate

$$f_{sh} = M \cdot f_s \quad (1)$$

where M is the oversampling ratio of $u(m)$ over $x(n)$.

The over sampled signal $u(m)$ is separated into section of length N_w by multiplying the rectangular window $w(m)$. The i -th finite duration sequence $\hat{u}_i(m)$ is then filtered by a low pass filter (LPF), whose passband is $0 \leq f \leq f_b$. The LPF eliminates the unnecessary frequency components and retains only the baseband signal within $0 \leq f \leq f_b$. The filter output $v_i(m)$ is an interpolated version of $\hat{u}_i(m)$ and has a sequence of length N_v :

$$N_v = N_w + N_h - 1 \quad (2)$$

where N_h is LPF length. By extracting successive $N_s (= N_v/M)$ samples of $v_i(m)$, we obtain $\hat{v}_i(m)$. Next,

$\hat{v}_i(m)$ is coded by a predictive coder and produces the coder output $y(m)$ of length N_s .

B. DECODER

In the decoder, input signal $y(m)$ is converted to $\hat{v}_i^*(m)$ by a predictive decoder. $\hat{v}_i^*(m)$ is a replica of $\hat{v}_i(m)$ in the coder. Since $v_i(m)$ in the coder is band limited within $0 \leq f \leq f_b$, it is possible to reconstruct $v_i^*(m)$ of length N_v from $\hat{v}_i^*(m)$ of length N_s by extrapolation using their linear dependent relations based on the discrete Fourier transform (DFT). Though $v_i^*(m)$, reconstructed version of $v_i(m)$, is of length N_v , the final decoder output is needed only at every M -th sample, because the sampling rate at the decoder output is f_s . Hence, we need calculate $v_i^*(m)$ only at $m = M \cdot n$ to obtain $\hat{v}_i^*(n)$. $\hat{v}_i^*(n)$ is of length N_s , and has a overlapping portion with the next section $\hat{v}_{i+1}^*(n)$. Final decoder output $x(n)$ is obtained by overlapping-addition of the adjacent $\hat{v}_i^*(n)$'s.

III. DESIGN CONSIDERATION

This section discusses the design of each functional block in detail.

A. LOW PASS FILTER

An LPF is used to band limit the zero interpolated signal $u(m)$. In order to extrapolate $v_i^*(m)$ in the decoder using the dependent relations of the DFT, the filter output $v_i(m)$ must be a finite duration sequence. Hence, the LPF must be a finite impulse response (FIR) filter with passband $0 - f_b$ Hz and a sampling frequency $M \cdot f_s$ Hz. Required filter specifications for speech signal are:

passband : 0 - 3400 Hz
stopband : 4000 - 8000 x M Hz
passband ripple A_p : as low as possible
stopband attenuation A_s : as high as possible

The filter length N_h is roughly proportional to M for given A_p and A_s ^[2]. For $A_p = \pm 0.05$ dB and $A_s = 80$ dB, N_h is given by

$$N_h \approx 45M \quad (3)$$

Fig.3 illustrates a block diagram of the LPF. Since $\hat{u}_i(m)$ has non-zero values only every M -th samples, we need only N_h/M multiplications per output sample. Moreover, the LPF output $v_i(m)$ is required by only N_s samples, we can save computation by N_s/N_v .

B. PREDICTIVE CODER/DECODER

The transfer function $H_p(z)$ of the coder is chosen as

$$H_p(z) = (1 - z^{-1})^m, \quad 0 \leq m \leq K-1$$

$$= (1 - z^{-1})^K, \quad K \leq m \leq N_s-1 \quad (4)$$

where m is the time index of $\hat{v}_i(m)$ and K is a predictor order. This $H_p(z)$ gives a robust predictor in the sense that predictor coefficients are independent of the input signal statistics and the sampling frequency^[3]. The predictor order K is one of important design parameters. To evaluate the order K , let us consider the case where coder input is sinusoidal signal of frequency f_b , which is the most severe frequency among all of in-band frequency components.

Let the coder input signal be

$$\hat{v}_i(m) = \cos(m\omega_b + \phi) \quad (5)$$

where $\omega_b = 2\pi f_b/Mf_s = \pi/M$.

Then, the output $y(m)$ of the coder $H_p(z)$ is

$$y(m) = A^{(m)} \cos(m\omega_b + \phi^{(m)}), \quad 0 \leq m \leq K-1$$

$$= A^{(K)} \cos(m\omega_b + \phi^{(m)}), \quad K \leq m \leq N_s-1 \quad (6)$$

where $A^{(m)} = (2\sin(\pi/2M))^m$ and $\phi^{(m)}$ is constant, function of m and ω_b . Hence, the magnitude of $y(m)$ decreases as K is increased. Let w_i be a word length of the input $\hat{v}_i(m)$ with sign bit, then the required word length $w_0(m)$ for the output of m -th order predictive coder including sign bit is

$$2^{(w_0(m)-1)} = (2\sin(\pi/2M))^m / 2^{-(w_i-1)}$$

So

$$w_0(m) = m \log_2(2\sin(\pi/2M)) + w_i \quad (7)$$

K is chosen so that $w_0(K)$ in (7) is equal to or less than 1.

The total number of bits to represent $\hat{v}_i(m)$ of length N_s is given by

$$N_T = \sum_{m=0}^{K-1} w_0(m) + N_s - K$$

$$= Kw_i + \frac{K(K-1)}{2} \log_2(2\sin(\pi/2M)) + (N_s - K) \quad (8)$$

Circuit configuration of the predictive coder and decoder are shown in Fig.4. In this figure, a predictor $P_m(z)$ is given by

$$P_m(z) = 1 - H_p(z)$$

$$= -\sum_{k=1}^m (-1)^k \binom{m}{k} z^{-k}, \quad 0 \leq m \leq K-1$$

$$= -\sum_{k=1}^K (-1)^k \binom{K}{k} z^{-k}, \quad K \leq m \leq N_s-1 \quad (9)$$

where $\binom{m}{k}$ is the binomial coefficient.

C. EXTRAPOLATOR

Band limited signal can be reconstructed from a part of original signal samples using an extrapolation technique^[4]. Let the DFT of $v_i(m)$ be $V_i(\ell)$. Since the $v_i(m)$ is band limited within $0 \leq f \leq f_b$ by the LPF,

$$V_i(\ell) = \sum_{m=0}^{N_v-1} v_i(m) \exp(-j2\pi \ell m/N_v) = 0, \quad \pi/M \leq 2\pi \ell/N_v \leq \pi \quad (10)$$

Rewriting real and imaginary components separately,

$$\sum_{m=0}^{N_v-1} v_i(m) \cos(2\pi \ell m/N_v) = 0$$

$$\sum_{m=0}^{N_v-1} v_i(m) \sin(2\pi \ell m/N_v) = 0 \quad \pi/M \leq 2\pi \ell/N_v \leq \pi \quad (11)$$

Eqn.(11) gives $(M-1)N_v/M$ linear equations with N_v variables. Hence, if N_v/M elements of $v_i(m)$'s are known, we can solve the equations about the remaining $(M-1)N_v/M$ $v_i(m)$'s. By a matrix representation, the unknown $v_i(m)$'s are given by

$$\underline{v}_u = \underline{c} \cdot \underline{v}_k \quad (12)$$

where \underline{v}_u : column $(M-1)N_v/M$ - vector of unknown $v_i(m)$'s,

\underline{v}_k : column N_v/M - vector of known $v_i(m)$'s,

\underline{c} : $(M-1)N_v/M \times N_v/M$ coefficients matrix.

Each entry of the matrix \underline{c} can be calculated from the DFT coefficients $\exp(-j2\pi \ell m/N_v)$ if the time index m of known $v_i(m)$'s are specified.

Since the final decoder output is needed only at every M -th point, only $v_i^*(m)$, $m = n \cdot M$, $0 \leq n \leq N_s-1$, is sufficient to be computed in the extrapolator.

IV. BIT-RATE COMPRESSION RATIO

The bit-rate compression ratio ρ is defined as the ratio of bit-rate at the coder output over the input signal bit-rate. Let the word length of the input $x(n)$ be w_i , then the bit-rate of input signal is

$$B(\text{input}) = w_i \cdot f_s \text{ bits/sec.} \quad (13)$$

From the previous discussion, the coder output requires N_T bits to transmit $y(m)$ in every N_w/f_{sh} second, then the bit-rate of the output is

$$B(\text{output}) = N_T/(N_w/f_{sh}) = Mf_s N_T/N_w \text{ bits/sec.} \quad (14)$$

Assuming $v_i(m)$ in the coder has the same word length as that of input $x(n)$, the compression ratio ρ is given by

$$\rho = \frac{B(\text{output})}{B(\text{input})} = MN_T/w_i N_w \quad (15)$$

Numerical examples for ρ are shown in Fig.5 and parameter values are tabulated in Table 1 and 2. As shown in Fig.5, the proposed method can reduce the bit-rate to 1/5 - 1/8 compared to linear PCM coding. Required number of multiplications per second $O(\text{MPL})$ is given by

$$O(\text{MPL}) = f_{sh} (NhN_s/MN_w + 2NsK/N_w + N_s(N_v - N_s)/MN_w) \approx f_s N_v (Nh/M + 2K + N_v/M)/N_w \quad (16)$$

$O(\text{MPL})$ for the example in Fig.5 a) is around 280 $\times f_s$ for each case. This number is acceptable for actual hardware implementation.

V. NOISE EVALUATION

Noise sources in the proposed method are:

- (1) Finite stopband attenuation of LPF
 - (2) Quantization error in the predictive coder
 - (3) Extrapolation error
 - (4) Round-off error in the LPF computation.
- (4) can be neglected if the signal word length in LPF is appropriately chosen.

Signal components in the stopband of the LPF fall into baseband by $M:1$ decimation. This noise can be expressed as

$$N_1 = (M-1)10^{-As/10} \frac{1}{2\pi} \int_{-\pi/M}^{\pi/M} |U(e^{j\omega})|^2 d\omega \quad (17)$$

Next, quantization error in the coder is

$$N_2 = 2^{-2(w_i-1)}/12 \quad (18)$$

where w_i is the word length of the coder input.

In eqn. (11), infinite loss of A_s is assumed. However, this assumption is not actually satisfied and the extrapolation causes error to $v_i^*(m)$. Let the extrapolation error be $r(m)$, then

$$\sum_{m=0}^{Nv-1} r(m) \exp(-j2\pi \ell m/Nv) = V(\ell) \quad (19)$$

$$= H(\ell) U(\ell)$$

where $H(\ell)$ is an LPF transfer function and $U(\ell)$ is the DFT of $u(m)$.

By taking squared-summation over ℓ from $Nv/2M$ to $Nv-Nv/2M-1$,

$$\frac{M-1}{M} Nv \sum_{m=0}^{Nv-1} r^2(m) = 10^{-As/10} \sum_{\ell=Nv/2M}^{Nv-Nv/2M-1} |U(\ell)|^2 \quad (20)$$

$$\ell = Nv/2M$$

Hence, the extrapolation error is given by

$$N_3 = \sum_{m=0}^{Nv-1} r^2(m) \approx M 10^{-As/10} \frac{1}{2\pi} \int_{-\pi/M}^{\pi/M} |U(e^{j\omega})|^2 d\omega \quad (21)$$

Total noise power N is

$$N = N_1 + N_2 + N_3$$

$$= (2M-1)10^{-As/10} \frac{1}{2\pi} \int_{-\pi/M}^{\pi/M} |U(e^{j\omega})|^2 d\omega + 2^{-2(w_i-1)}/12 \quad (22)$$

Suppose the coder input is a sine-wave with amplitude A . The signal to noise ratio (SNR) for the input is given by

$$\text{SNR} = 10 \log \left(\frac{A^2}{2} / \left(\frac{A^2}{2} (2M-1) 10^{-As/10} + 2^{-2(w_i-1)}/12 \right) \right) \quad (23)$$

Fig. 6 shows the SNR characteristics for $A_s = 70\text{dB}$ and $w_i = 13\text{bits}$. As shown in Fig. 6, SNR is slightly degraded as M is increased, because the noise caused by finite stopband attenuation of the LPF becomes predominant for high level input.

VI. CONCLUSION

A new approach for low bit-rate coding is proposed. Numerical evaluation for the bit-rate compression ratio and SNR shows the proposed method provides good performance compared with the conventional linear PCM coding. Furthermore, since the predictor in the coder does not utilize any statistics of input signal, this method can be applied for both speech and music coding.

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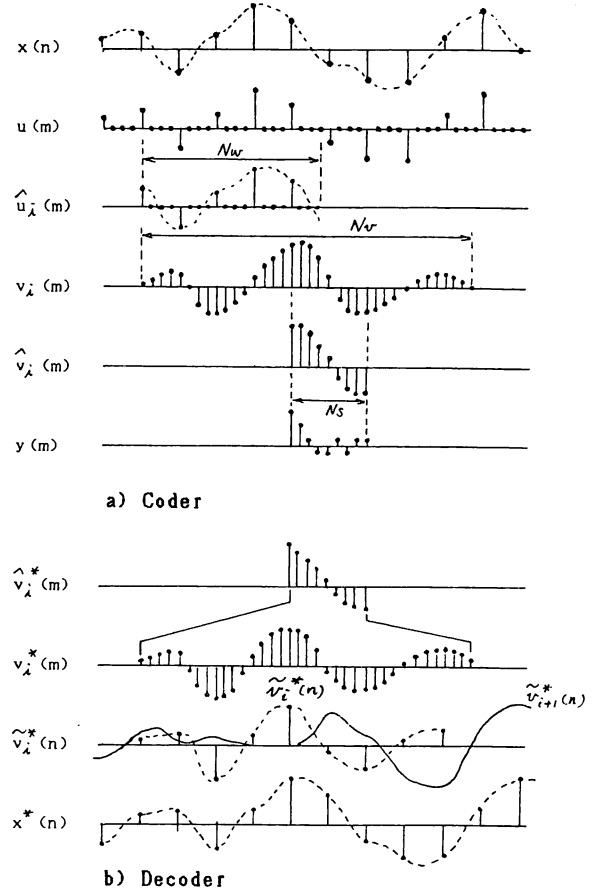


Fig. 2. Signal waveforms.

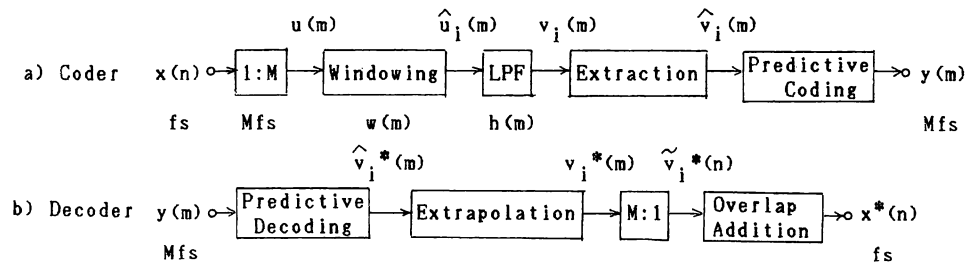
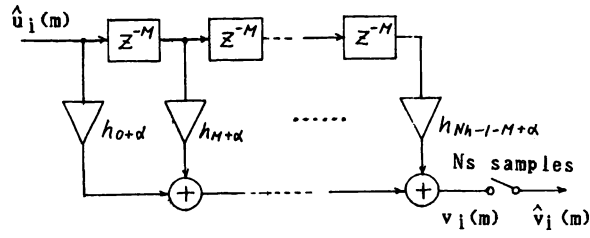


Fig. 1. Block diagram of coder/decoder.



$$\alpha = ((m))_M : m \text{ modulo } M$$

Fig. 3. LPF block diagram.

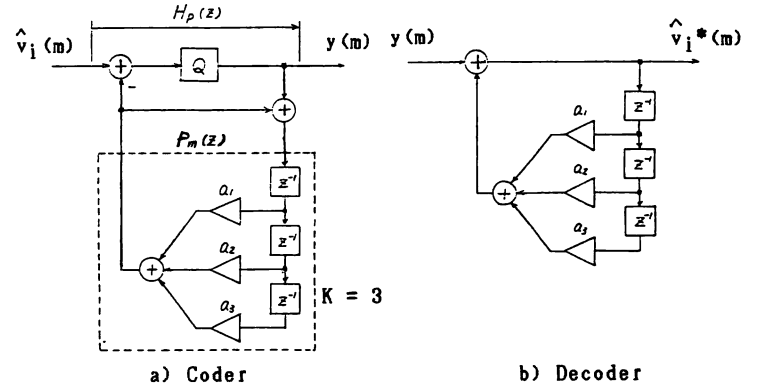


Fig. 4. Predictive coder/decoder.

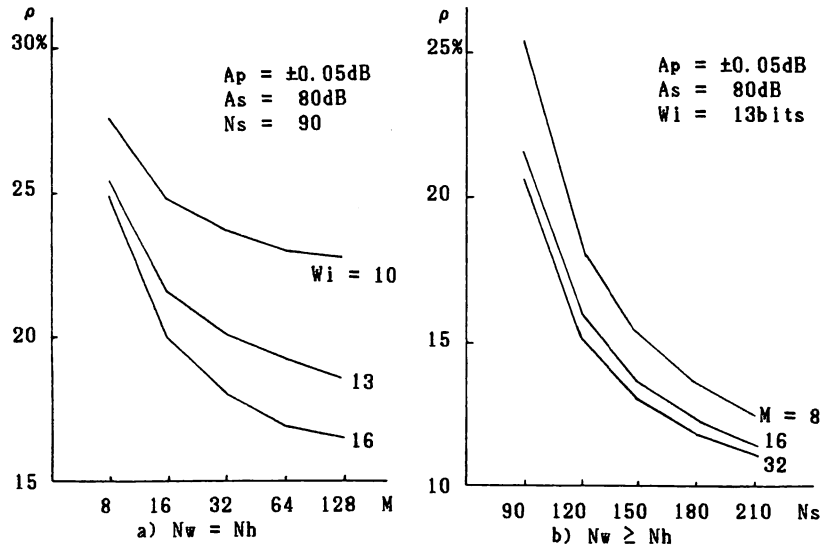


Fig. 5. Bit-rate compression ratio.

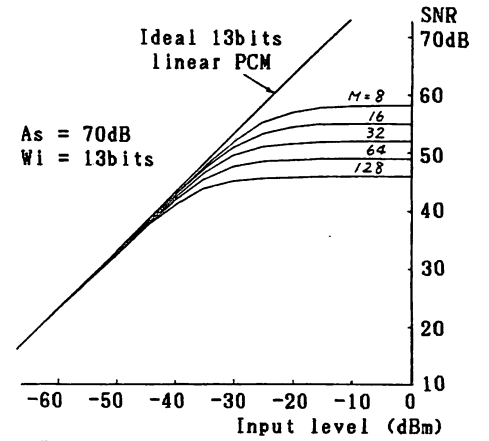


Fig. 6. SNR characteristics.

Table 1. Parameter values for example in Fig. 5 a).

M	Wi	K	Nw = Nh	Nv	Ns
8	10	7	360	719	90
	13	9			
	16	11			
16	10	4	720	1439	90
	13	5			
	16	6			
32	10	3	1440	2879	90
	13	4			
	16	4			
64	10	2	2880	5759	90
	13	3			
	16	3			
128	10	2	5760	11519	90
	13	2			
	16	3			

Table 2. Parameter values for example in Fig. 5 b).

M	Wi	K	Nh	Nw	Nv	Ns
8	13	9	360	360	719	90
				600	959	120
				840	1199	150
				1080	1439	180
				1320	1679	210
16	13	5	720	720	1439	90
				1200	1919	120
				1680	2399	150
				2160	2879	180
				2640	3359	210
32	13	4	1440	1440	2879	90
				2400	3839	120
				3360	4799	150
				4320	5759	180
				5280	6719	210